

6.5 Exponential Equations

Bellwork

Simplify.

1.  $\frac{t^6 u^3}{t^3 u^1} = t^{6-3} u^{3-1} = t^3 u^2$

2.  $\frac{g^7 h^3 m}{h^6} = g^{7-6} h^{3-1} m = g^1 h^2 m$

3.  $\left(\frac{3a^2 b^6}{2}\right)^3 = \frac{3^3 a^6 b^{18}}{2^3} = \frac{27a^6 b^{18}}{8}$

4.  $\left(\frac{f^{-4} g^3}{h^{-5}}\right)^{-1} = \frac{f^{-4} g^3}{h^{-5}}$

5.  $\frac{m^3 p^3}{mp} = m^{3-1} p^{3-1} = m^2 p^2$

6.  $\frac{c^5 d^4 f^4}{gd^6 f^2} = \frac{c^{5-1} f^{4-2}}{d^{5-3}} = \frac{c^4 f^2}{d^2}$

Warm Up 1-3

**Core Concept**

**Property of Equality for Exponential Equations**

**Words** Two powers with the same positive base  $b$ , where  $b \neq 1$ , are equal if and only if their exponents are equal.

**Numbers** If  $2^x = 2^5$ , then  $x = 5$ . If  $x = 5$ , then  $2^x = 2^5$ .

**Algebra** If  $b > 0$  and  $b \neq 1$ , then  $b^x = b^y$  if and only if  $x = y$ .

Solve each equation.

a.  $3^{x+1} = 3^5$

$$\begin{array}{r} x+1 = 5 \\ -1 \quad -1 \\ \hline x = 4 \end{array}$$

b.  $6^1 = 6^{2x-3}$

$$\begin{array}{r} 1 = 2x - 3 \\ +3 \quad +3 \\ \hline 4 = 2x \\ \frac{4}{2} = \frac{2x}{2} \\ 2 = x \end{array}$$

c.  $10^{3x} = 10^{2x+3}$

$$\begin{array}{r} 3x = 2x + 3 \\ -2x \quad -2x \\ \hline x = 3 \end{array}$$

Example 1

Solve the equation. Check your solution.

1.  $2^{2x} = 2^6$

$$\begin{array}{r} 2x = 6 \\ \frac{2x}{2} = \frac{6}{2} \\ x = 3 \end{array}$$

2.  $5^{2x} = 5^{x+1}$

$$\begin{array}{r} 2x = x + 1 \\ -x \quad -x \\ \hline x = 1 \end{array}$$

3.  $7^{3x+5} = 7^{x+1}$

$$\begin{array}{r} 3x + 5 = x + 1 \\ -x \quad -x \\ \hline 2x + 5 = 1 \\ -5 \quad -5 \\ \hline 2x = -4 \\ \frac{2x}{2} = \frac{-4}{2} \\ x = -2 \end{array}$$

Monitoring Progress 1-3

Solve

(a)  $5^x = 125$

$5^x = 5^3$

$x = 3$

(b)  $4^x = 2^{x-3}$

$(2^2)^x = 2^{x-3}$

$2^{2x} = 2^{x-3}$

$2x = x - 3$

$-x = -3$

$x = -3$

(c)  $9^{x+2} = 27^x$

$3^{2(x+2)} = 3^{3x}$

$2x + 4 = 3x$

$-2x = -4$

$4 = x$

Example 2

Solve

(a)  $\left(\frac{1}{2}\right)^x = 4$

$(2^{-1})^x = 2^2$

$2^{-x} = 2^2$

$-x = 2$

$x = -2$

(b)  $4^{x+1} = \frac{1}{64}$

$4^{x+1} = 4^{-1}$

$2^{2(x+1)} = 2^{6(-1)}$

$2x + 2 = -6$

$-2 = -8$

$x = -4$

Example 3

$2x = -8$

$x = -4$

Solve the equation. Check your solution.

4.  $4^x = 256$

$2^{2(x)} = 2^8$

$2x = 8$

$x = 4$

6.  $4^{3x} = 8^{x+1}$

$2^{2(3x)} = 2^{3(x+1)}$

$2^{6x} = 2^{3x+3}$

$6x = 3x + 3$

$3x = 3$

$x = 1$

5.  $9^{2x} = 3^{x-6}$

$3^{2(2x)} = 3^{x-6}$

$4x = x - 6$

7.  $\left(\frac{1}{3}\right)^{x-1} = 27$

$3^{-1(x-1)} = 3^3$

$3^{-x+1} = 3^3$

$-x + 1 = 3$

$-x = 2$

$4x = x - 6$

$3x = -6$

$x = -2$

Monitoring Progress 4-7

Homework

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