9.3 Solving Quadratics (Square Root and Factoring)

Bellwork

Find the solution of the system:

$5x + 2y = 9$

$x = -y - 3$

Remember solving by graphing is finding the x-intercepts

$5(-y-3)+2y=9$

$-5y-15+2y=9$

$-3y-15=9$

$+15 \quad +15$

$\frac{-3y}{-3} = \frac{24}{-3}$

$y = -8$

$x = -y - 3$

$x = -(-8) - 3$

$= 8 - 3$

$x = 5$

$x = 5$, $y = -8$ $(5, -8)$

Solve by factoring.

1. $w^3 - 8w^2 + 16w = 0$

$w(w^2 - 8w + 16) = 0$

$a \cdot c = 16$

-1	-16
-2	-8
-4	-4

$(w^2 - 4w)(4w + 16) = 0$

$w(w-4) - 4(w-4) = 0$

$w(w-4)(w-4) = 0$

$w = 0$ $\quad w - 4 = 0$ $\quad w - 4 = 0$

$+4 \quad +4 \quad +4 \quad +4$

$w = 4 \quad w = 4$

2. $x^2 - 25 = 0$

$x^2 + 0x - 25 = 0$

$a \cdot c = -25$

1	-25
5	-5

$(x^2 + 5x)(5x - 25) = 0$

$x(x+5) - 5(x+5) = 0$

$(x-5)(x+5) = 0$

$x - 5 = 0 \quad x + 5 = 0$

$x = 5 \quad x = -5$

Solve by factoring.

1. $8x^2 - 56x + 48 = 0$

$8(x^2 - 7x + 6) = 0$

$a \cdot c = 6$: $-1 \cdot -6$, $2 \cdot 3$

$(x^2 - 1x)(6x + 6) = 0$

$x(x-1) - 6(x-1) = 0$

$8(x-6)(x-1) = 0$

$8 \neq 0$ $\quad x-6=0$ $\quad x-1=0$

$\boxed{x=6}$ $\quad \boxed{x=1}$

2. $14x^2 + 31x = -15$

$14x^2 + 31x + 15 = 0$

$a \cdot c = 210$

1	210
2	105
3	70
5	42
6	35
7	30
10	21

$(14x^2 + 10x) + (21x + 15) = 0$

$2x(7x+5) + 3(7x+5) = 0$

$(2x+3)(7x+5) = 0$

$2x+3=0$ $\quad 7x+5=0$

$\boxed{x = -\frac{3}{2}}$ $\quad \boxed{x = -\frac{5}{7}}$

3. $2x^2 + 5 = 7x$

$2x^2 - 7x + 5 = 0$

$a \cdot c = 10$: $-1 \cdot -10$, $-2 \cdot -5$

$(2x^2 - 2x)(-5x + 5) = 0$

$2x(x-1) - 5(x-1) = 0$

$(2x-5)(x-1) = 0$

$2x-5=0$ $\quad x-1=0$

$\boxed{x = \frac{5}{2}}$ $\quad \boxed{x=1}$

4. $3x^2 - 14x + 8 = 0$

$a \cdot c = 24$

-1	-24
-2	-12
-3	-8
-4	-6

$(3x^2 - 2x)(-12x + 8) = 0$

$x(3x-2) - 4(3x-2) = 0$

$(x-4)(3x-2) = 0$

$x-4=0$ $\quad 3x-2=0$

$\boxed{x=4}$ $\quad \boxed{x = \frac{2}{3}}$

Steps to solve by square root:

First, isolate x^2 on one side of the equation. Then solve by taking the square root of each side.

Solutions to $x^2 = d$:

* If d is positive, there are two solutions
* If d is negative, there are no real solutions (cannot square root negative #'s)
* If d is zero, there is one solution ($\sqrt{0} = 0$)

ex. $\sqrt{x^2} = \sqrt{16}$

$x = \pm 4$

$x = 4$ $\quad x = -4$

b/c $(4)^2 = 16$

$(-4)^2 = 16$

Solving Quadratics using Square Roots:

Solve each.

1. $3x^2 - 27 = 0$

$+27 \quad +27$

$\frac{3x^2}{3} = \frac{27}{3}$

$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$

$x = 3$

$x = -3$

2 solns

2. $-5x^2 + 11 = 16$

$-11 \quad -11$

$\frac{-5x^2}{-5} = \frac{5}{-5}$

$\sqrt{x^2} = \sqrt{-1}$

$x = \sqrt{-1}$

No real solution, cannot $\sqrt{}$ negative #'s

3. $x^2 - 10 = -10$

$+10 \quad +10$

$\sqrt{x^2} = \sqrt{0}$

$x = 0$

1 solution

4. $\sqrt{(x-1)^2} = \sqrt{25}$

$x - 1 = \pm 5$

$x - 1 = 5 \quad x - 1 = -5$

$+1 \quad +1 \quad +1 \quad +1$

$x = 6 \quad x = -4$

2 solutions

5. $-3x^2 = -75$

$\frac{-3}{-3} \quad \frac{-75}{-3}$

$\sqrt{x^2} = \sqrt{25}$

$x = \pm 5$

$x = 5 \quad x = -5$

2 solutions

6. $x^2 + 12 = 10$

$-12 \quad -12$

$\sqrt{x^2} = \sqrt{-2}$

$x = \sqrt{-2}$

No real solution

7. $\sqrt{(x+7)^2} = \sqrt{0}$

$x + 7 = 0$

$-7 \quad -7$

$x = -7$

1 solution

8. $\frac{4(x-3)^2}{4} = \frac{64}{4}$

$\sqrt{(x-3)^2} = \sqrt{16}$

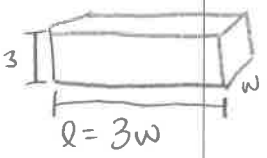
$x - 3 = \pm 4$

$x - 3 = 4 \quad x - 3 = -4$

$+3 \quad +3 \quad +3 \quad +3$

$x = 7 \quad x = -1$

A touch tank has a height of 3 feet. Its length is three times its width. The volume of the tank is 270 cubic feet. Find the length and width of the tank to the nearest tenth.



$V = \ell \cdot w \cdot h$

$270 = 3w(w)(3)$

$270 = 9w^2$

$\frac{270}{9} = \frac{9w^2}{9}$

$\sqrt{30} = \sqrt{w^2}$

$\pm\sqrt{30} = w$

$+\sqrt{30} = w$

cannot have a negative width

$h = 3$

$\ell = 3w$

$w = w$

$V = 270$

$w = 5.5$ feet

$\ell = 3(5.5)$

$= 16.4$ feet

A storage container has the shape of a rectangular prism. Its height is 6 feet. its length is two times its width. The volume is 288 cubic feet. Find the length and width of the container to the nearest tenth.

$V = \ell \cdot w \cdot h$

$288 = 2w(w)(6)$

$\frac{288}{12} = \frac{12w^2}{12}$

$\sqrt{24} = \sqrt{w^2}$

$\pm\sqrt{24} = w$

$4.9 = w$

$h = 6$

$\ell = 2w$

$w = w$

$V = 288$

$w =$ 4.9 feet

$\ell = 2(4.9) =$ 9.8 feet

Homework

9.3 Pg. 501 #4 - 30 even, 33, 34

~~14~~, 24 28 ~~8~~